Palingram Artworks:

A System of Graphic Design and Descriptive Geometry

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Introduction

In the latter part of 2004 and early part of 2005, I made a series of artworks based on an original design which I first conceived and gave finished form to in 1976 in a limited edition of screenprints with the title *Palingenesis* (a word coined by the Italian philosopher, Vincenzo Gioberti (d. 1852), which means "... the return of human concepts to the essential centre of being from which they become divorced. "[1] hence, a kind of "re-incarnation of ideas." According to the Merriam-Webster on-line dictionary, the word "palingenesis" is composed of a Greek word "palin", meaning "again" and the Latin word "genesis," meaning "birth," hence the idea of "re-incarnation." I have invented the word "palingram" to describe my recent artworks, as they employ the repeated use of tangent circles, and "palin" represents this fundamental repetition.

Several months earlier, I had received a request for permission to use my on-line image of *Palingenesis* as the "opening image" for the September, 2004 issue of the "e-zine" *Episteme, An International Journal of Science, History and Philosophy*, published by the Department of Computer Science, University of Via Vanvitelli, located in Perugia, Italy. This request "catalyzed" me to "re-create," hence, to "re-incarnate" the original artwork, *Palingenesis*, or similar artworks in similar or different media. To accompany these activities, I thought it would be an interesting challenge to be able to verbally describe the elements and principles involved in the creation of "palingram artworks," and it is this "descriptive geometry" which is attempted in the present paper.

1.0 Definition of a "palingram"

A "palingram" is a geometrical object composed of "nested palin-dots," a palin-dot being geometrically equivalent to a circle, and "nesting" referring to the placement of a small palin-dot tangent to and "within the confines of" a larger palin-dot (likewise, one might view this same object as a larger palin-dot "surrounding" the smaller of the two, and tangent to it). Palingrams composed of 2, 3, 4 and 5 nested palin-dots are also known as (a.k.a.) "du-ali," "tri-ali," "quadr-ali," "pent-ali," respectively.

1.1 Assignment of names to "palingram artworks"

The term "palingram artwork" will be given only to geometrical arrangements the elements of which are limited to a given variant of "shaded pent-ali" and/or their "anti-shaded" counterpart(s) (see 1.2.3 and 1.5.4 below). A unique palingram artwork (different from other palingrams, even though it may be one of many similar or identical copies) has a special name or title to distinguish it from other palingram artworks, e.g., *Palinbloom, Palincompass, Palinbalance, Palingate, Palindream, Cross-palination, Palinjazz, Palinduet, At the Palinball,* etc. Admittedly whimsical in nature, the name is intended to reflect some intrinsic quality of the design of each artwork.

1.2 Definition of "palin-pair" or "du-ali" (pronounced "do-a-lie")

A palin pair or du-ali is "an alignment of two 'palin-dots' along an 'axis of alignment'" (see 1.2.1, below). As can be seen from Fig. 1, below, a palin-pair is formed by 2 tangent circles (each of which is individually referred to as a "palin-dot"), *the larger of which encloses the smaller and the smaller of which is enclosed by the larger.* Axiomatically, the diameter of the smaller circle is set equal to or greater than the radius of the larger circle. A palin-pair has several characteristics:

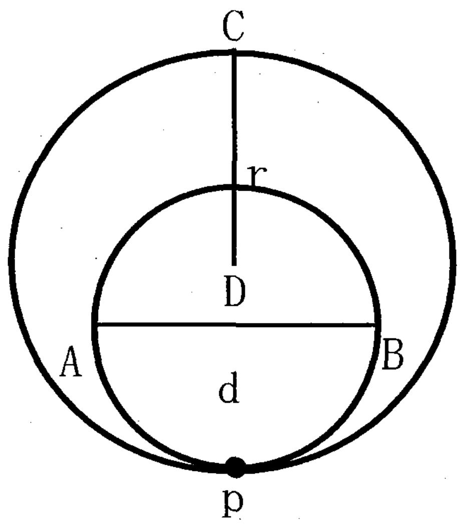


Fig. 1: Basic palin-pair

1) the two palin-dots are tangent at point "p";
2) as an axiomatic restriction, d (AB) of smaller circle must be equal to or greater than r (CD) of the larger circle, thus,

$$d \geqq r \quad \text{(axiom 1)}$$

1.2.1 Definition of the "axis of alignment"

In any given palin-pair, the line that includes the centers (x) of both palin-dots shall be called the "axis of alignment" of the palin-pair. See Fig. 2, below:

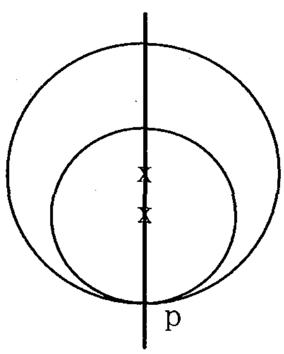


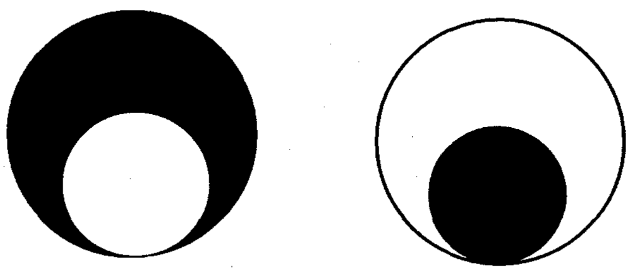
Fig. 2: Axis of alignment of a palin-pair

The centers of both palin-dots lie on the axis of alignment; the axis of alignment passes through p, the point of tangency. There is an axis of alignment for higher-order "palin-nests" (see 1.3, below), as well.

1.2.2 The area enclosed within the du-ali is divided into two: 1) a circular area equal to the smaller palin-dot, and: 2) a second area that has a circular perimeter equal to the circumference of the larger palin-dot and a shape that has two "diminishing arms" that appear to "embrace" the smaller palin-dot. These "embracing arms" may also be viewed as a kind of "crescent" shape, with the two arms meeting, theoretically, in the single point of tangency. This latter description is more easily visualized when "shading" is applied. See 1.2.3 and Fig. 3 below:

1.2.3 Shading/Anti-shading of a Du-ali

Only one palin-dot in a du-ali may be shaded, in either of two configurations, as shown in Figs. 3A and 3B below:



A B

Fig. 3: Two possible shadings of a du-ali

With reference to each other, each palin-pair is referred to as the "anti-shading" of the other. In A above, the "crescent arms" are shown as shaded in black; in B, they appear to be white.

1.2.4 Du-ali Symmetry

If the viewer assumes that the objects here described are first drawn on an (invisible) Cartesian grid in order to establish "up", "down", "left" and "right" directions, etc., on the visual plane, it then becomes possible to describe alternative du-ali, e.g., where the white palin-dot of in Fig. 4A is moved vertically along the Y-axis to a point of tangency at the very "top" of the black palin-dot; or, again, where the black palin-dot in Fig. 4B, is rolled along the inner perimeter of the larger palin-dot to any of 360 different points of tangency. The results, in both of these cases, would be equivalent to a rotation of the original object with respect to the center of the larger palin-dot, and does not result in a new configuration. Thus, we can say that (axiom 2) *du-ali (and higher order objects) are symmetric with regard to rotation.* In the discussion that follows, we will disallow merely symmetrical replications of an object due to rotation, and allow only those transpositions of palin-dots or palin-nests which occur along the axis of alignment, and, with respect to the shape as a whole, result in new configurations.

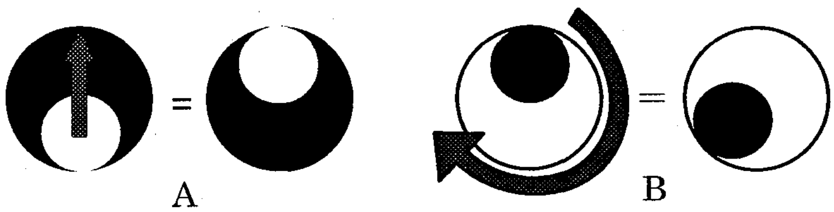


Fig. 4: Rotationally symmetric du-ali

1.3 Definition of 2 types of unshaded 3-dot palin nests ("tri-ali"): "knule" and "klune"

(Axiom 3) *Palin-dots can be "nested,"* e.g., a small palin-pair can be tangentially enclosed in a larger palin-dot, or can tangentially enclose an even smaller palin-dot to form a "3-dot palin-nest." As long as the restrictions regarding the relative lengths of diameter and radius are maintained and if the centers of all three palin-dots lie along a single axis of alignment, the object thus formed is called a "nested tri-alignment," abbreviated as "tri-ali" (pronounced "try-a-lie"). Tri-ali (I am here adopting a "Latin plural," as in "alumni," even though it is identical to the singular, hence, 1 tri-ali, 2 tri-ali, etc.) come in two forms, one having a single point of tangency (1-p) shared by the three palin-dots, and a second form having two separate points of tangency (2-p), each shared by two palin-dots, as shown in Figs. 5A and 5B, below:

A: (3)1-p tri-ali—a.ka. a "knule"

B: (2/2)2-p triali—a.k.a. a "klune"

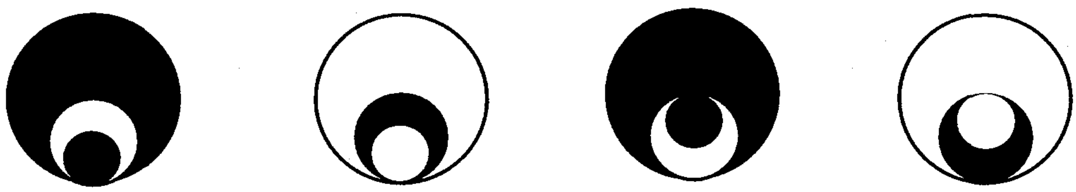
Fig. 5: Two forms of un-shaded tri-ali

In figures 5A and 5B, above, A is a 3-dot palin-nest with a single-point of tangency shared by all 3 palin-dots, which may be designated as a [(3)1-p tri-ali]; B is a 3-dot palin-nest with two points of tangency, which may be designated as a [(2/2)2-p tri-ali] .

The number(s) in parentheses indicate(s) the number of palin-dots sharing(a) point(s) of tangency. While awkward to read, these designations, by differing one from another, convey at a glance that the objects they designate are, likewise, unique, and not simply rotated versions of one another. For the sake of convenience, form A shall arbitrarily be called a “knule” (pronounced “newl”), and form B shall arbitrarily be called a “klune” (pronounced “kloon”).

1.3.1 Four configurations of shaded tri-ali

If three or more palin-dots are “nested”, shading can occur on an “odd” or “even” basis, i.e., *with the inner-most palin-dot as the “start point” for numeration*, then palin-dots 1, 3, 5…n may be shaded, while 2, 4, 6…n+1 remain un-shaded (or vice versa, to produce the anti-shading). See figures 6A, 6B, 6C, 6D below:



A: Odd-basis (1,3) shading of a knule, a.k.a. a “black knule”

B: Even-basis (2) shading of a knule, a.k.a. a “white knule”

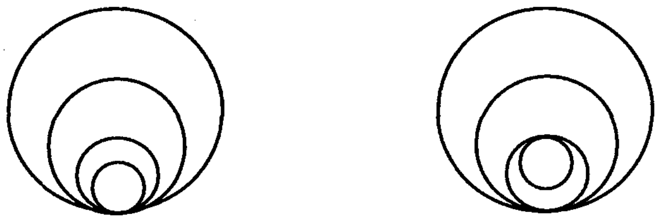
C: Odd-basis (1,3) shading of a klune, a.k.a. a “black klune”

D: Even-basis (2) shading of a klune, a.k.a. a “white klune”

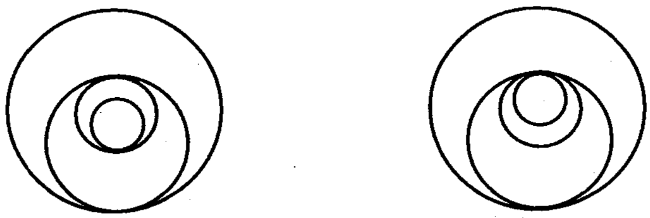
Fig. 6: Odd-basis and even-basis shading/anti-shading of tri-ali

1.40 Definition of 4 types of unshaded 4-dot palin-nests ("quadr-ali")

There are 4 forms of unshaded 4-dot palin-nests (a.k.a. "quadr-ali"), each with a unique form of tangency (as observed from the bottom upwards), as can be verified by their differing alpha-numerical designations in Figs.7A, 7B, 7C, and 7D below:



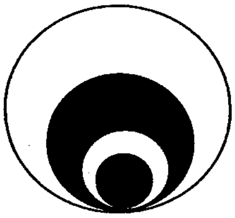
A: (4)1-p quadr-ali B: (3/2)2-p quadr-ali



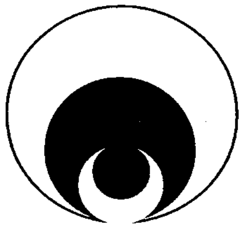
C: (2/2/2)3-p quadr-ali D: (2/3)2-p quadr-ali

Fig. 7: 4 forms of unshaded quadr-ali

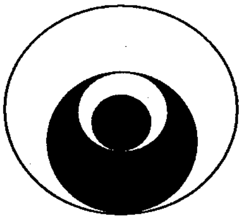
1.4.1 To better visualize the forms of quadr-ali, we can shade/anti-shade them according to the principles described in 1.3.1, above. Note: with the addition of shading, it appears that a quadr-ali is either a knule or a klune which is "embraced by a pair of crescent arms", and that, with reference to the latter, has either an "upwards" orientation or a "downwards" orientation, hence the additional verbal descriptions which accompanying figures 8A, 8B, 8C and 8D (also, 8E, 8F, 8G and 8H) below:



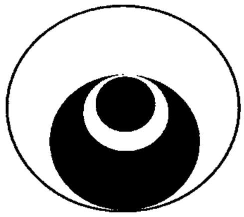
A: (4)1-p quadr-ali,
a.k.a."a downward
black knule embraced by
white crescent arms"



B: (3/2)2-p quadr-ali,
a.k.a."a downward
black knule embraced by
white crescent arms"



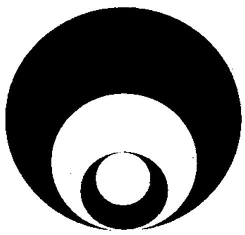
C: (2/2/2)3-p quadr-ali,
a.k.a "an upward
black klune
embraced by white
crescent arms"



D: (2/3)2-p quadr-ali,
a.k.a."an upward
black knule
embraced by white
crescent arms"

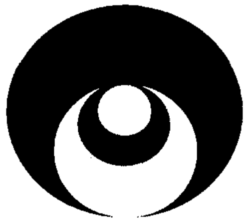
Figs. 8A - 8D: 4 forms of shaded quadr-ali

E: (4)1-p quadr-ali,
a.k.a. "a downward
white knule
embraced by black
crescent arms"



F: (3/2)2-p quadr-ali,
a.k.a. "a downward
white klune
embraced by black
crescent arms"

G: (2/2/2)3-p quadr-ali, a.k.a. "an upward white klune embraced by black crescent arms"



H: (2/3)2-p quadr-ali, a.k.a. "an upward white knule embraced by black crescent arms"

Fig. 8E-8H: 4 forms of anti-shaded quadr-ali

1.50 Definition of 8 types of unshaded 5-dot palin-nests ("pent-ali")

Obviously, the principles of construction elaborated so far may be carried out *ad infinitum*, but both visualization and nomenclature become increasingly cumbersome as the number of palin-dots increases.

1.5.1 For our present purposes, however, it is sufficient to carry out the extension of the principles here described only one additional "order" to that of 5-dot palin-nests, or "pent-ali" (pronounced "pent-a-lie"). The next figure will help us to better visualize the transition from the order of quadr-ali to that of pent-ali, along with the shaded and anti-shaded configurations of the latter (see figures 9A – 9C, below):

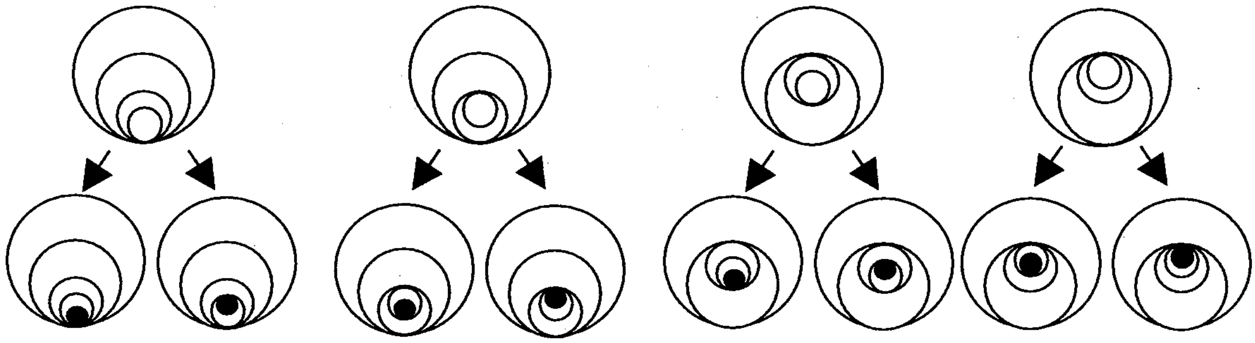


Fig. 9A: Doubling of 4 types of quadr-ali and addition of 1 shaded palin-dot to make 8 partially-shaded pent-ali

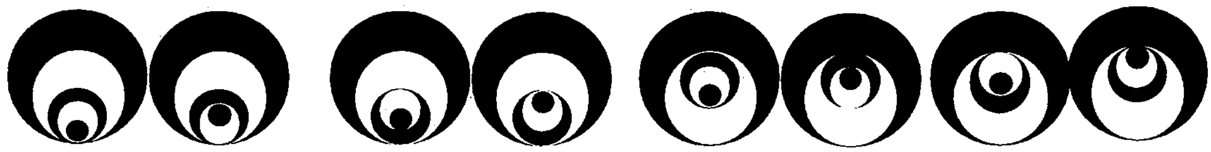


Fig. 9B: 8 types of (odd-basis) shaded pent-ali

Fig. 9C: 8 types of (even-basis) anti-shaded pent-ali

Figs. 9A - 9C: Visualization of transition from quadr-ali to pent-ali

Here it can be pointed out at that a simple mathematical formula governs the relationship between the number of palin-dots (n)and the total number of unshaded palingram forms (k) which, following the rules of construction, can be made from them.

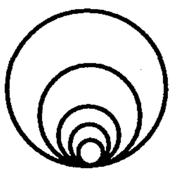
For n = 2 or n = greater than 2, the combined number unshaded palingram forms (k) may be calculated by

$$k = 2^{n-1}/2$$

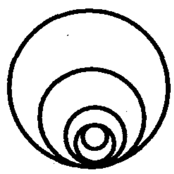
Thus, for the first 4 “orders” of palingrams beyond the palin-dot, the following table may be completed:

n	$k = 2^{n-1}/2 =$ unshaded variations	2k=shaded/anti-shaded variations
2 (palin-pair)	$2^{2-1} = 2^1 \Rightarrow 2/2 = 1$	2
3 (tri-ali)	$2^{3-1} = 2^2 \Rightarrow 4/2 = 2$	4
4 (quadr-ali)	$2^{4-1} = 2^3 \Rightarrow 8/2 = 4$	8
5 (pent-ali)	$2^{5-1} = 2^4 \Rightarrow 16/2 = 8$	16

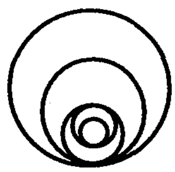
1.5.2 To verify that each of the 8 unshaded forms of pent-ali are unique, we need merely look at the alpha-numerical designations of the unshaded forms, as shown in Figs. 10A - 10H, below:



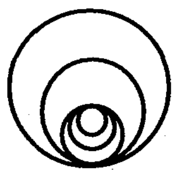
A: (5)1-p pent-ali



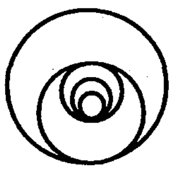
B: (4/2)2-p pent-ali



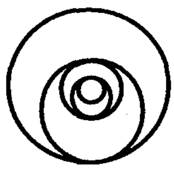
C: (3/2/2)3-p pent-ali



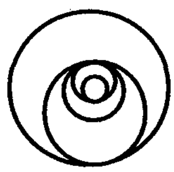
D: (3/3)2-p pent-ali



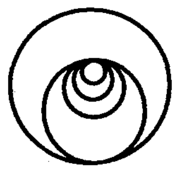
E: (2/3/2)3-p pent-ali



F: (2/2/2/2)4-p pent-ali



G: (2/2/3)3-p pent-ali

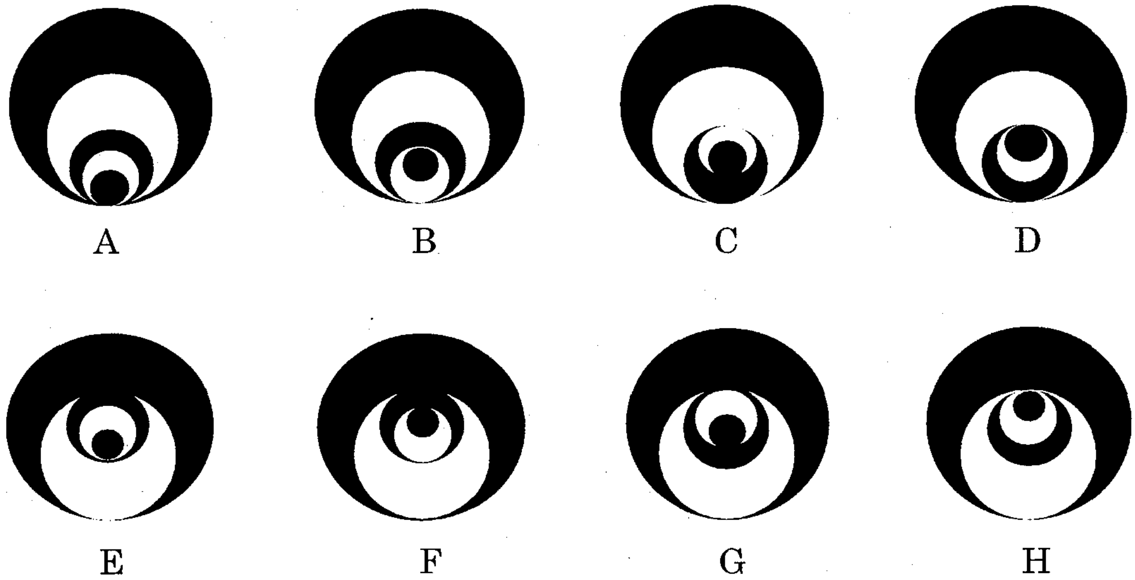


H: (2/4)2-p pent-ali

Figs. 10A - 10H: 8 forms of unshaded pent-ali with their unique alpha-numerical designations

The fact that each of the above alpha-numerical designations is unique is proof that this is the complete set of pent-ali and that none of them are simply symmetrical rotations of others.

1.5.2 A rational nomenclature of pent-ali poses considerable difficulties (see Figs. 11A – 11H, below). The object in Fig.11A appears to be “a small downward black knule, overlayed onto/embedded within a larger downward white knule, embraced by a pair of downward black crescent arms”; that of Fig. 11B appears to be “a small downward black klune, overlayed onto/embedded within a larger downward white knule, embraced by a pair of downward black crescent arms”; Fig. 11C appears to be “an small upward black klune, overlayed onto/embedded within a larger downward white klune, embraced by a pair of downward black crescent arms”; Fig 11D appears to be a small upward black knule, overlayed onto/embedded within a larger downward white klune, embraced by a pair of downward black crescent arms; Fig.11E appears to be “a small downward black knule, overlayed onto/embedded within a larger upward white klune, embraced by downward black cresent arms”; Fig. 11F appears to be a small downward black klune, overlayed onto/embedded within a larger upward white klune, embraced by downward black crescent arms”; Fig.11G appears to be “a small upward black klune overlayed onto/embedded within a larger upward white knule, embraced by downward black crescent arms; Fig. 11H appears to be “a small upward black knule overlayed onto/embedded within a larger upward white knule, embraced by downward black crescent arms.” Similar names could be assigned to the 8 anti-shaded variants of pent-ali.



Figs. 11A - 11 H: 8 forms of shaded pent-ali

1.5.4 Self-similarity.

Of all of the configurations above, two, in particular, possess the property of "self-similarity", which refers to the way that additional palin-dots create patterns similar to those already existing. In Figs. 12A, and 12B. below (shaded, so as to make the pattern more easily perceived) additional palin-dots created additional overlayed/embedded knules (12A), or, additional palin-dots created additional overlayed/embedded klunes (although the direction, upward or downward, appears to reverse in an alternating sequence, e.g., downwards, upwards, downwards) (12B). Of course, the same can be said for the anti-shaded versions of these objects, although the visual appearance would be somewhat different.

A: (5)1-p pent-ali

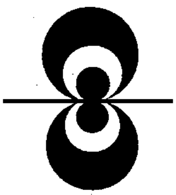
B: (2/2/2/2)4-p pent-ali

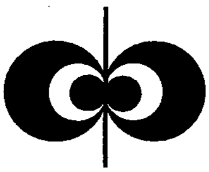
Figs. 12A and 12B: Self-similarity in two specific type of pent-ali

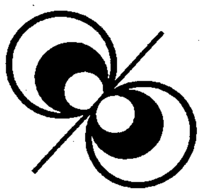
1.5.5 While all of the various shaded and anti-shaded pent-ali have some qualities of visual interest, *for the purpose of "palingram artworks", the form shown in Fig. 12B, above (and its anti-shaded version), may be regarded as the ones most visually interesting and intriguing because of their reversing, self-similarity.* All the others may, for the present purpose, be ignored.

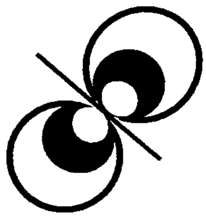
2. Mirror images

For every shaded or anti-shaded du-ali, tri-ali, quadr-ali, and pent-ali there exists its "mirror-image" with which it may be paired along one of 4 "mirror axes," viz., horizontal, vertical, postively-sloped "diagonal" or negatively-sloped "diagonal". See Figs. 13A, 13B, 13C, 13D below:









A B C D

Fig. 13A: Odd-shaded tri-ali with its mirror image on the opposite side of a horizontal mirror axis

Fig. 13B: Odd-shaded tri-ali with its mirror image on the opposite side of a vertical mirror axis

Fig. 13C: Even-shaded tri-ali pair aligned on opposite sides of a 45-degree positive slope diagonal mirror axis

Fig. 13D: Mirror image pair of even-shaded tri-ali aligned along opposite sides of a negative 45-degree slope diagonal mirror axis

Fig. 13A - 13D

2. (2/2/2/2)4-p Pent-ali Groupings on the Cartesian plane

(2/2/2/2)4-p pent-ali (see Fig.12B in 1.5.4 above), having reversing, self-similarity, may be grouped in groups of 4 around the origin of the Cartesian plane in such a way that one pent-ali is situated in each quadrant of the plane. Pairing a pent-ali with its mirror image in an adjacent or cater-corner quadrant, or with its anti-shading variant results in a number of interesting and aesthetically pleasing images, as shown in Figs. 14A, 14B, 14C, below. Because of their aesthetically pleasing complexity, these groupings are the first such arrangements to be called or "palingram artworks". Each unique arrangement has been given a unique name to help distinguish it from others (see 1.1, above).

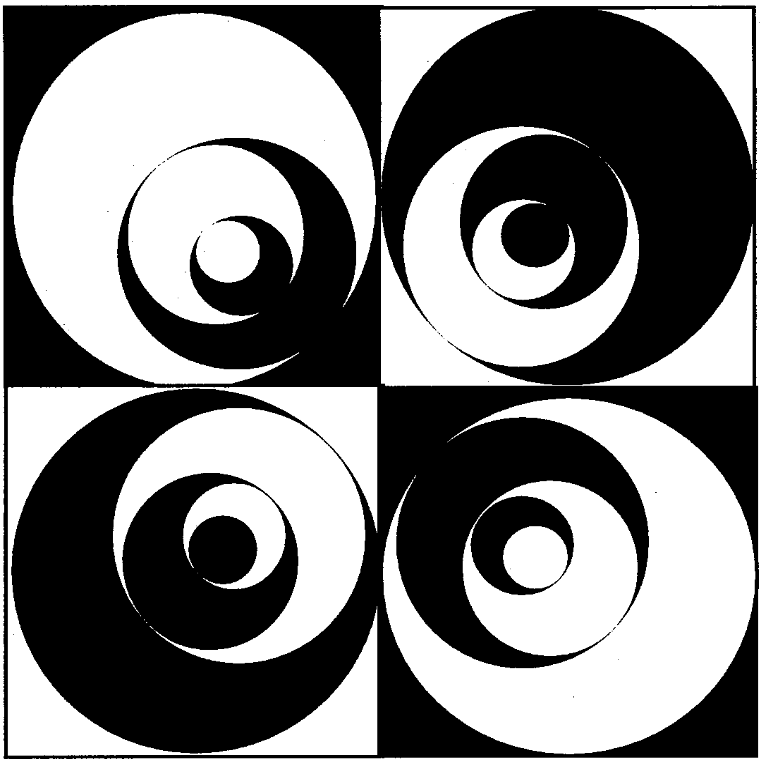


Fig. 14A Palingram: *Palinbloom*

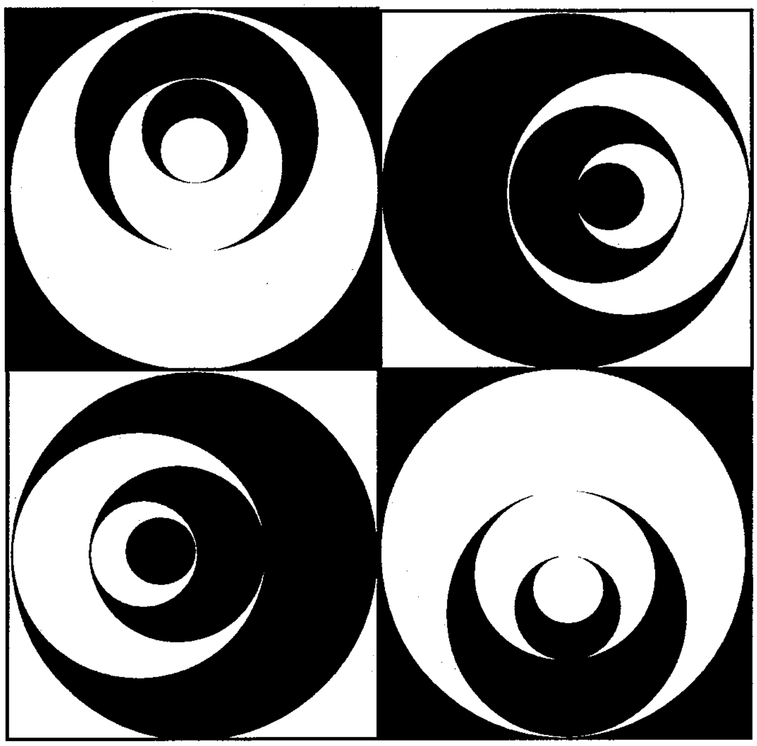
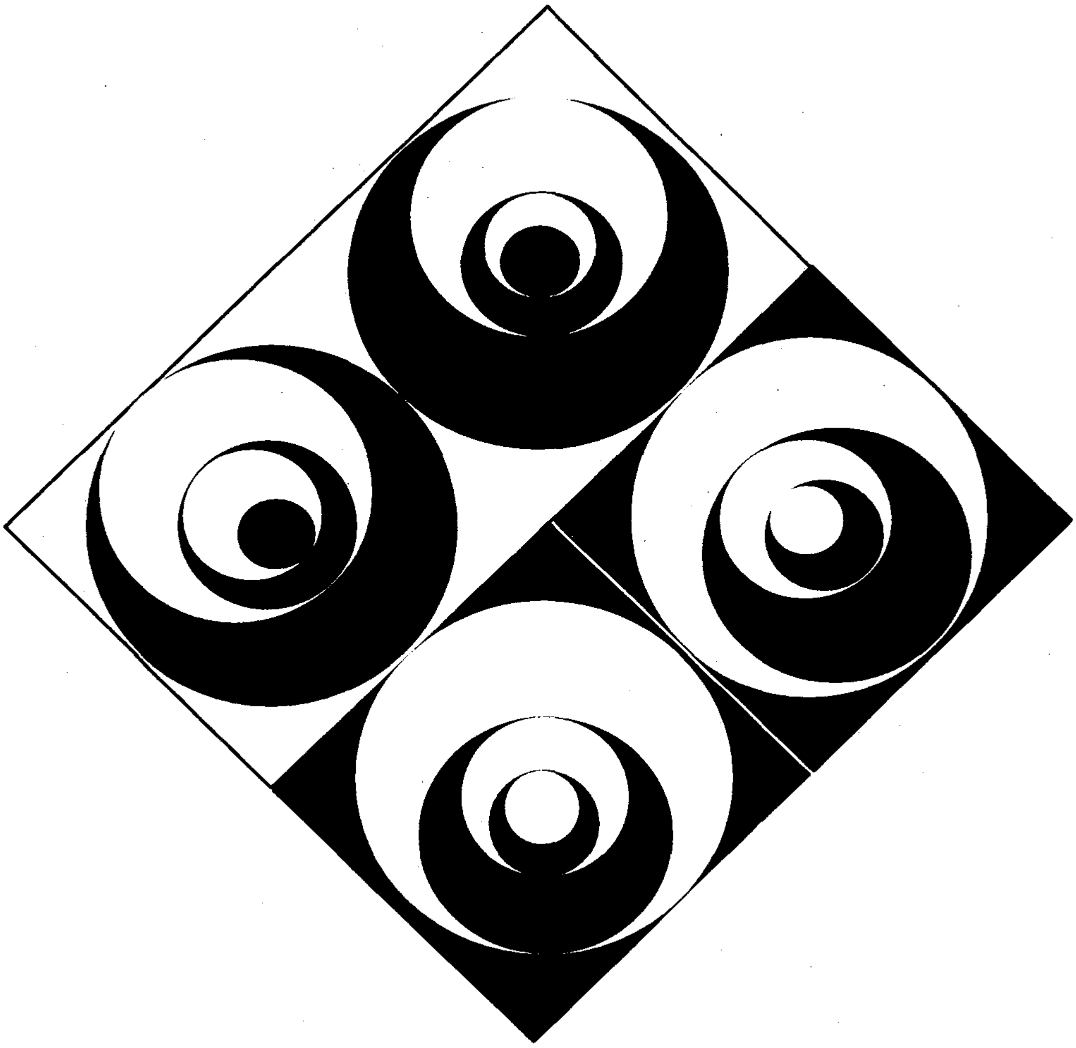


Fig 14B Palingram: *Palincompass*



14C Palingram: *Palingenesis*

For additional examples of palingram artworks, some of which incorporate color, please visit the author's web site: "Hal Slovic's Virtual Gallery of Original Artworks" at the following URL: http://www.haslovart.com and follow the links beginning at [*Palinbloom 2004.04*], [About this work], ["Palingram Artworks"]

References:

[1]"Gioberti, Vincenzo" *Encyclopedia Britannica* from Encyclopedia Britannica Premium Service.
http://www.britannica.com/eb/article?tocId=9036878
[Accessed November 23, 2004]

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